Planar, univalent σ-harmonic mappings

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A mapping $U = (u^1, u^2) : \Omega \subseteq R^2 \to R^2$ is called harmonic if its components are harmonic functions. The most well known example is given by a holoporphic mapping. Injective holomorphic mappings are the usual conformal mappings. Harmonic mappings which are not necessarily holomorphic but injective have been studied by Radó, Knneser and Choquet, who exhibited a class of Dirichlet conditions for harmonic mappings which ensure global injectivity.

Our talk will review work of the past few years in collaboration with G. Alessandrini. Our focus is on σ-harmonic mappings. Its component are $W^{1,2}_{\mathrm{loc}}(\Omega; R^2)$ weak solutions to the p.d.e.

$$\mathrm{div}(\sigma(x)\nabla u(x)) = 0$$

The matrix σ is only assumed to be uniformly elliptic and measurable with L^∞ entries. Particular emphasis is placed on the case where $\Omega \equiv R^2$ and σ is periodic of some unit cell.

If time permits, we will present work by the author outlining the importance of these results in the contests both of the theory of composite materials and of the area distortion problems in quasiconformal mappings.