Metamaterials and the Landau-Lifshitz Permeability Argument: Large Permittivity Begets High-Frequency Magnetism

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Homogeneous composites, or metamaterials, made of dielectric or metallic particles are known to show magnetic properties that contradict arguments by Landau and Lifshitz [*Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1960), p. 251] indicating that the magnetization and, thus, the permeability loses its meaning at relatively low frequencies. Here we show that these arguments do not apply to composites made of substances with $\kappa_S = \mathrm{Im}\sqrt{\varepsilon_S} >> \lambda / \ell$ or $n_S = \mathrm{Re}\sqrt{\varepsilon_S} \sim \lambda / \ell$ (ε_S and ℓ are the complex permittivity and the characteristic length of the particles, κ_S and n_S are, respectively, the extinction coefficient and the refractive index, and $\lambda >> \ell$ is the vacuum wavelength). Our general analysis is supported by studies of split-rings, one of the most common constituents of electromagnetic metamaterials, and spherical inclusions. An analytical solution is given to the problem of scattering by a small and thin split ring of arbitrary permittivity. Results reveal a close relationship between ε_S and the dynamic magnetic properties of metamaterials. For $\left|\sqrt{\varepsilon_S}\right| << \lambda / a$ (a is the ring cross-sectional radius), the composites exhibit very weak magnetic activity, consistent with the Landau-Lifshitz argument and similar to that of molecular crystals. In contrast, large values of the permittivity lead to strong diamagnetic or paramagnetic behavior characterized by susceptibilities whose magnitude is significantly larger than that of natural substances.

The double constraint $\kappa_S >> \lambda / \ell >> 1$ (or, $n_S \sim \lambda / \ell >> 1$ if $\kappa_S << n_S$) poses severe limitations for attaining magnetism at arbitrarily high frequencies. Because they have a large extinction coefficient, metals are to be favored at optical frequencies. Since $\varepsilon_S \approx -\omega_P^2 / \omega^2$ for $\omega\tau >> 1$, the constraint becomes $\lambda >> \ell >> \lambda_P$. The measured values of the permittivity for noble metals indicate that magnetism can coexist with the effective-medium condition for frequencies up to ~ 1.5×10^{14} Hz ($\lambda \sim 2.5$ μm).