

UMPSC
Fall 2023

problem set 3

Notes: You may use any resources to solve these problems. Make sure you understand your solution thoroughly if you submit for credit. Good luck!

Problem 1

Part A. Solve Laplace's equation on the disk. The solutions you find are harmonic functions, and satisfy very important averaging properties. Explain them.

Part B. Solve Poisson's equation on the disk.

Problem 2

Choose some interesting boundary conditions and source function, and then code a numerical solution to Poisson's equation on the disk. If this is too difficult, do Laplace's equation instead.

Problem 3

Part A. Prove the Riemann mapping theorem. This theorem is very important for numerically solving Poisson's equation on arbitrary simply connected domains.

Part B. You are given a regular hexagon in the complex plane, with some interesting boundary conditions (you come up with them). With the same source function as you used in problem 2, use the Schwarz Christoffel formula to solve Poisson's equation on a hexagon.