

UNIVERSITY OF UTAH MATH CONTEST



PSET 1 SPRING 2024

POSTED: 2/2 at noon

DUE: 2/16 at noon

submit pdf of solutions to:

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Problem 1: A Problem from the Past

You are living in the 14th century and have never met L'Hopital. In fact, L'Hopital does not yet exist.

Find

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$$

Problem 2: What's Your Favorite Shape?

After solving Problem 1, you suddenly have an epiphany and remember all of calculus. That very same night, after winning a few games of pool, you're sitting at the pub with your mate Joe Edwards, and you ask him about his favorite shape. He looks at you, suprised for a moment by the question, and then lets out a toothy grin:

"My favorite shape is the shape with the most area given a fixed perimeter."

Joe replies calmly, and lets out a cackled laugh. So what is Joe's favorite shape? More importantly, why??

Problem 3: The Ill-Lit Casino

Part (a).

You go to sleep and wake up back in 2024. Your friends invite you to Las Vegas to celebrate your birthday. Upon arriving, you find an ill-lit casino with a strange offer:

Flip a coin. Heads, you win 1\$. Tails, you lose 1\$.

Simple enough. Stubborn as you are, you tell your friends you will not stop playing the game until you make a total net profit of 1\$. Assuming the coin is fair, and you can go into any amount of debt, how long should your friends expect to wait for you to finish playing the game?

Part (b).

After talking this over with your friends, Billy Jane Rae tells you that they heard the coin is NOT fair, but in fact there is only a 48% chance of getting heads and a 52% chance of tails. You shake your head in slight disapproval, but there is no look of surprise on your face. After all, this is Vegas.

Learning that the game is not perfectly fair, you decide you are not willing to go into any debt besides losing the cash you brought on hand. In your wallet there is currently 200\$, all one dollar bills.

After walking to the counter and obtaining a coin, Quanda Ratiio, the owner of the casino, tells you that you are allowed to gamble as much money as you want on a single coin flip (not just 1\$ per flip), with the same rules: heads, you win that much money. Tails, you lose that much money. Smiling with confidence, you choose to gamble at least 100\$ total in coin flips before you leave the counter. What is your best betting strategy?

(for example, one could bet 1\$ each flip for 100 flips, or 5\$ each flip for 20 flips, or 50\$ each flip for 2 flips. More creative strategies are allowed.)

Part (c).

After playing with the casino using the optimal betting strategy, you managed to not lose all of your money. You walk to the Aria and play poker for a while with your friends. Afterwards, you and your friends stop by an ill-lit pawn shop which sells a perfectly fair coin. Your friend Parsley Mariete immediately purchases the coin, and challenges you to a similar game. But this time, its back to 1\$ bets only. Staring you down with fiery eyes, Parsley echoes:

Flip a coin. Heads, you win 1\$. Tails, you lose 1\$.

Both of you are fierce competitors, and will not stop playing the game until you lose all of the cash currently in your wallet. Parsley takes out their wallet, and you do the same. After winning big in poker at the Aria, both you and Parsley each have 1500\$ cash in your wallets, still all one dollar bills. How long should you expect to play Parsley Mariete until one of you throws in the towel with an empty wallet?