# PROBLEM 3: THE CIRCLE GAME

## UNDERGRADUATE PROBLEM SOLVING CONTEST

Due Friday, December 4, 2009 by 5:00 PM

This problem has three parts. A correct answer to part (a) will be considered to be a partially correct solution; correct answers to parts (a) and (b) will be considered to be a fully correct solution. Part (c) will only be used as a tiebreaker in determining the problem winner.

(a) Fifty people are playing a game in which they sit in a circle, numbered in order from person #1 to person #50. The person who is directing the game starts with person #1 and counts, "1, 2, 1, 2, 1, 2, ..."; each person who receives a "2" loses and exits the circle immediately. For example, at the start, people #2, 4, 6 will exit the circle. The counting continues until there is only one person left. This person wins. What is his number?

(b) Suppose there are $n$ people playing the game, numbered in order from person #1 to person #$n$. How can you determine the number of the person who wins? (This will be a very simple algorithm involving $n$.)

(c) If there are $n$ people playing the game and the person directing the game counts, "1, 2, ..., $p$, 1, 2, ..., $p$, ...", and every $p$th person is removed, how can you determine the number of the person who wins? Provide a recursive method involving $n$ and $p$ that would allow you to determine who wins when $n$ people play if you know who wins when $n-1$ people play.

In the spirit of the UPSC, you should not search the internet or look the solution up in a book.