PROBLEM 1: TILING A CHECKERBOARD
UNDERGRADUATE PROBLEM SOLVING CONTEST
DUE SEPTEMBER 24, 2008 BY 5:00 PM

Suppose you have a 7×7 checkerboard (grid of squares, colored with black and red, no two squares of the same color share an edge) and dominoes the size of two adjacent squares on the board. Suppose one corner is black. A tiling of the checkerboard with dominoes will be defined by every square being covered by precisely one domino and no dominoes hang over the edge of the board. (Each of the following questions implicitly includes "If so, why and how? If not, why not?")

1. (Easy) Is it possible to tile the entire board?

2. (Less Easy) If we remove one corner, is it possible to tile the board?

3. (Still not exactly hard) What if we remove a red square sharing an edge with a corner? Can we tile the board?

4. (Now it's a contest) Precisely which single cells can be removed from the checkerboard to leave a tilable checkerboard?

The appropriate citation for this problem will accompany the next problem.