Problem 3: Sum of Perfect Squares

Undergraduate Problem Solving Contest

Due November 20, 2008 by 5:00 pm

If $n = m^2$ where m is an integer, we call n a perfect square.
For example 9 is a perfect square since $9 = 3^2$.

Primary Question: If n is a positive integer such that $2n + 1$ is a perfect square, show that $n + 1$ is the sum of two successive perfect squares.

Tie Breaker: If n is a positive integer such that $3n + 1$ is a perfect square, show that $n + 1$ is the sum of three perfect squares.

A correct solution to the primary problem is a fully correct solution. The tie breaker will only be used in deciding the overall winner.